Workbook on
Data Collection, Analysis and Validation

January 19-21, 2021

STATISTICAL INDIAN INSTITUTE

भिन्नेष्वैक्यस्य दर्शनम्

UNITY IN DIVERSITY

Sponsored by

cpcb

Indian Statistical Institute,

7, S. J. S Sansanwal Marg, New Delhi-110 016

Workbook on
Data Collection, Analysis and Validation

PREFACE

This course is an introduction to ***Statistical Methods for Data Collection, Analysis and Validation*** and it aims to provide you to carry out compilation, statistical analysis, data interpretation of simple data sets and an exposure to use commonly available computer software packages. The development of such packages has made it much easier for everyone to apply statistics to their observations of the world.

This workbook contains a set of exercises. Most of the exercises will be based on calculations that can be done without the aid of a computer, the answers to these problems will be discussed in the tutorial sessions.

After attending this course, we hope that the participants will be able to:

- do some basic statistical analysis of data.
- have the confidence to apply the techniques studied and perhaps slightly more advanced techniques.

S.K. Neogy

January 19, 2021 *Professor & Course Coordinator*

EXERCISE 1

Vehicles of different types were moving on the Prithviraj Road of Delhi, primarily two wheelers, cars and buses. Emissions of the two wheelers, cars and buses were examined for particulate matter, SO_2 and CO. About 20 vehicles of each type was checked. Design a check sheet for collecting data.

EXERCISE 2

Surface Water contamination is a serious issue. Enumerate the possible causes for this contamination through a Cause and Effect Diagram.

EXERCISE 3

Methyl tertiary Butyl Ether (MTBE) is a chemical compound mixed with gasoline for proper combustion, and is highly soluble in water. It is carcinogenic in nature. Surface water generally has MTBE dissolved in it, which makes it unfit for drinking purposes. Kolkata has lots of ponds, presence of MTBE is a major hazard. Data are collected from 40 ponds for the quantity of MTBE in these ponds in parts per billion (ppb) units.

38	68	50	32	44	25	49	57
46	48	40	47	36	48	52	44
68	26	38	76	63	19	54	65
46	73	42	47	35	53	40	35
61	45	35	42	50	56	45	28

- Construct a frequency distribution
- Draw the Histogram
- Compute Mean, Median, Quartiles and Interquartile range.
- Compute Standard Deviation, Variance and Coefficient of Variation.

Worksheet for Exercise 3

Class Interval	Tally Mark	Frequency	Mid point			

Mean = $\frac{\quad}{40}$ =

Median : (i) Arrange data in increasing order

Median =

First Quartile =

Third Quartile =

InterQuartile Range = Q_3 - Q_1 =

Standard deviation = $\sqrt{\dfrac{\sum (x-\bar{x})^2 f}{n-1}}$

Coefficient of Variation =

EXERCISE 4

Assume the same situation and reconsider the data in Exercise 3 in samples of size 2 as follows:

Sample No.	1	2	Mean	Sample No.	1	2	Mean
1	38	46		11	44	36	
2	68	46		12	63	35	
3	61	68		13	50	25	
4	48	26		14	48	19	
5	73	45		15	53	56	
6	50	40		16	49	52	
7	38	42		17	54	40	
8	35	32		18	45	57	
9	47	76		19	44	65	
10	47	42		20	35	28	

1) Calculate mean for each sample and overall mean of all the twenty sample means.
2) Make a frequency distribution of the sample means.
3) Calculate the Standard deviation and variance from these 20 sample means and compare with the corresponding values obtained in Exercise 3.

Worksheet for Exercise 4

Class Interval	Tally Mark	Frequency	Mid Point			

Mean = $\frac{}{20}$ =

Standard deviation = $\sqrt{\frac{\sum (X - \overline{X})^2 f}{n-1}}$

Coefficient of variation =

Your Conclusion regarding frequency distribution, mean and Standard Deviation

1. **---**
2. **---**
3. **---**

EXERCISE 5

Samples of ambient air from 60 towns was found to have mean Carbon Monoxide concentration as 5 mg / m^3 with a standard deviation of 0.40 mg/m^3 for an hour observation. Calculate

1) % of towns that will have CO concentration more than 5.8 mg/m^3
2) Proportion of towns having CO concentration less than 4.6 mg/m^3
3) % of towns having CO concentration between 4.4 and 5.6 mg/m^3

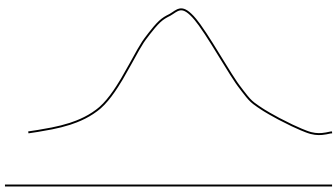
Worksheet for Exercise 5

Solution : X = Concentration of CO in a town

μ = σ =

(a) Mark the required area

Z value for X = 5.8 mg /m^3, $Z = \frac{X - \mu}{\sigma}$ =

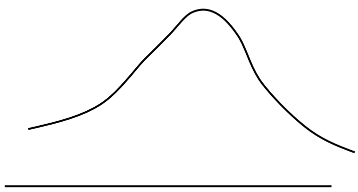


Area below Z from table A =

% of towns having concentration of CO over 5.8 mg /m^3 =

(b) Mark the required area

Z value for X = 4.6 mg /m^3, $Z = \frac{X - \mu}{\sigma}$ =



Area below Z from table A =

% of towns having concentration of CO more than 4.6 mg /m^3 =

(c) % of towns having concentration more than 4.4 and 5.6 mg /m^3 =

EXERCISE 6

1. In an air pollution study the following amounts of suspended benzene soluble organic matter (in microgram per cubic meter) were obtained at an experiment station for eight different samples of air. 2.2, 1.8, 3.1, 2.0, 2.4, 2.0, 2.1 and 1.2 . Construct a 99% confidence interval for the corresponding true mean.

2. You know that water contains pathogens and chemicals which makes it unfit for drinking purposes. Ten samples of water are taken from different areas and the number of days after which the samples become turbid is noted down counted before it became turbid. Construct a 95% confidence interval for the turbidity to appear. Data is as follows :

   8, 24, 24, 6, 15, 35, 63, 59, 34, 39 .

3. The Shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained. Construct 95% confidence level for the true mean shelf life.

108 days	135 days
124	163
124	159
106	134
115	139

Worksheet for Exercise 6

Solution :

1. Confidence interval is given by

$$\overline{X} \pm \frac{t_\alpha}{2} \frac{s}{\sqrt{n}}$$

Sample Mean ($\overline{X}$) =

Sample Standard Deviation(s) = $\sqrt{\frac{\sum\left(x-\overline{x}\right)^2}{n-1}}$ =

$\alpha/2$ = and degrees of freedom (d.f.) =

$\frac{t_\alpha}{2}$ = from t – distribution (Table B) with (d.f.) =

Hence the 99% confidence interval for the mean =

2. Confidence interval is given by

$$\overline{X} \pm \frac{t_\alpha}{2} \frac{s}{\sqrt{n}}$$

Sample Mean ($\overline{X}$) =

Sample Standard Deviation(s) = $\sqrt{\frac{\sum\left(x-\overline{x}\right)^2}{n-1}}$ =

$\alpha/2$ = and degrees of freedom (d.f.) =

$t_{\frac{\alpha}{2}}$ = from t – distribution (Table B) with (d.f.) =

Hence the 95% confidence interval for the mean =

3. Confidence interval is given by

$$\overline{X} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$Sample\ Mean\ (\overline{X}) =$,

$Sample\ Standard\ Deviation(s)\ \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n-1}} =$

$\frac{\alpha}{2} =$ $and\ degrees\ of\ freedom\ (d.f.) =$

$t_{\frac{\alpha}{2}} =$ $from\ t-distribution\ (Table\ B)\ with\ (d.f.) =$

Hence the 95% confidence interval for the true shelf life =

EXERCISE 7

Nine determinations were made by technician of MTBE with the following results in ppb.

26, 27, 26, 25, 26, 27, 26, 25

Test at 5% level of significance the hypothesis that the results are consistent with the published value of 20 ppb of MTBE permissible in British Columbia.

Worksheet for Exercise 7

Solution :

H_0 : (μ =) vs H_1 : (μ =)

Sample Mean ($\overline{X}$) = and sample Standard Deviation (s) =

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} =$$

For α = , value from t – distribution table =

.with degrees of freedom =

Your Decision : Reject or Accept H_0

Your Conclusion :

EXERCISE 8

Benzene is a compound found in water samples of urban areas. Four chemists are asked to determine the percentage of Benzene in water samples in three determinations and results are shown below :

Chemist	Percentage of Benzene		
1	84.99	84.04	84.38
2	85.15	85.13	84.88
3	84.72	84.48	85.16
4	84.20	84.10	84.55

(a) Do chemists differ significantly ? Use $\alpha = 0.05$

3. Obtain an estimate of repeatability and reproducibility variability.

4. Calculate repeatability and reproducibility limits.

Worksheet for Exercise 8

1. Find total for each chemist and grand total (G)

C_1 = C_2 = C_3 = C_4 =

And $G = C_1 + C_2 + C_3 + C_4$ =

2. Correction Factor C.F. = $\frac{G^2}{3 \times 4}$ =

3.Total Sum of Squares (TSS) = $(84.99)^2 + (8.40)^2 + \ldots\ldots\ldots.. + (84.55)^2$ - C.F.

=

4. Sum of Squares due to chemists (SSC) = $C_1^2 + C_2^2 + C_3^2 + C_4^2$ - C.F.

=

5.Sum of Squares due to within error (repeatability) SSE = TSS – SSC =

6. Breakup of Degrees of freedom. Degrees of freedom for

(i) TSS =

(ii) SSC =

(iii) SSE =

ANOVA

Source	d.f	Sum of Square	Mean Square	F
Chemist				
Error				
Total				

At α = 5%, F value from table with d.f. (,) =

Your Conclusion

Estimate of repeatability =

Estimate of chemist to chemist variability =

Precision =

EXERCISE 9

For measurement of concentration of lead in water, a standard solution with concentration of 1.00 mg/litre was prepared and tested for homogeneity. Stability test was done on 10 samples at the end of 45 days with the following concentration values.

Concentration of lead (mg/litre):

1.01, 1.01, 0.99 1.00, 0.99, 1.02, 1.01, 1.00, 0.99, 1.00

Test for the stability of the test material.

Worksheet for Exercise 9

Solution :

H_0 : (μ =) vs H_1 : (μ =)

Sample Mean ($\overline{X}$) = and Sample Standard Deviation (s) =

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} =$$

For α = , value from t – distribution table =

With degrees of freedom =

Your Decision : Accept or Reject H_0

Your Conclusion :

EXERCISE 10

The following data relate to the development of new condensation polymer. The characteristic is Basic NH_2 in 10^{-6} units. 11 samples were considered for homogeneity of the batch. Each sample was analysed three times . The results are given below :

Sample No.	Replicate 1	Replicate 2	Replicate 3	Total
A	64	70	63	
B	67	67	72	
C	60	68	67	
D	60	63	59	
E	56	60	56	
F	51	53	55	
G	57	58	62	
H	56	57	59	
I	59	58	59	
J	53	58	55	
K	56	63	60	

Test for the homogeneity of test material in the batch.

WORKSHEET FOR EXERCISE 10

Solution :

1. Find total for each sample

   A = B = C =

2. and grand total G = Sum of all reading =

3. Find correction factor =

4. Total sum of squares (TSS) =
   $(64)^2 + (70)^2 + \ldots\ldots\ldots + (60)^2$ - CF =

5. Sum of square between samples (SSB)

$$= \frac{S_A^2 + S_B^2 + \ldots\ldots + S_k,}{3} - CF$$

=

1. Sum of squares for error (repeatability)

= TSS – SSB =

WORKSHEET FOR EXERCISE 10

ANALYSIS OF VARIANCE TABLE

Source	Degree of Freedom	Sum of Squares	Mean Square	F. Ratio
Samples				
Error				
Total				

Mean Square = Sum of Squares / Degrees of freedom.

Degrees of Freedom for samples = No. of samples – 1.

Degrees of Freedom for Total = Total No. of Observations – 1.

Degrees of Freedom of Error

= No. of Samples * (No. of replicates – 1) =

F (permissible) is read from F table, for samples degrees of freedom as N_1 and Error Degrees of Freedom as N_2

EXERCISE 11

A Chemist wanted to study the effect of four chemicals on different types of air collected from different regions of the country. The Air Quality index was measured after the application of the chemical. The data relate to the experimental results :

Chemical Agent	Air type1	Air type2	Air type 3	Air type 4	Air type 5
A	73	68	74	71	67
B	73	67	75	72	70
C	75	68	78	73	68
D	73	71	75	75	69

Analyze the data suitability & find out whether chemical agents are different and air type are different ?

Worksheet for Exercise 11

Solution:

1. Find the total for each chemical agent and air type :

A = , B = , C = , D =

A_1 = , A_2 = , A_3 = , A_4 = , A_5 =

2. Grand Total (G) = Sum of all readings =

3. Correction Factor (C.F.) =

4. Total Sum of Squares (TSS) = $(73)^2 + (68)^2 + \ldots\ldots + (69)^2$ - C.F.

   =

5. Sum of Squares between Agents (SSA)

$$= \frac{A^2 + B^2 + C^2 + D^2}{5} - C.F =$$

6. Sum of Squares between air type (SSAT)

$$= \frac{A_1^2 + A_2^2 + A_3^2 + A_4^2 + A_5^2}{4} - C.\ F.\ =$$

7. Sum of Squares for error (SSE) = TSS –SSA – SSAT =

Worksheet for Exercise 11

ANALYSIS OF VARIANCE TABLE

Source	Degrees of freedom	Sum of Squares	Mean Sum of Squares	F Ratio
Chemical Agent				
Air Type				
Error				
Total				

Mean Sum of Square = Sum of Squares/ Degrees of freedom

Degrees of freedom for agents = Total no. of agents –1 =

Degrees of Freedom for air type = Total no. of air type – 1=

Degrees of freedom for total = Total no. of readings – 1 =

Degrees of freedom for error = D.f. for total – D.f. for agents – D.f. for air type =

F (permissible) is read from F table.

EXERCISE 12

Exercise on Analysis of AQI data :

As a part of calculating the AQI index , 25 samples of ambient air were sent to labs along with instruction to test the ozone concentration.

Sample Code No.	Ozone Concentration (X)	AQI Index (Y)
1	51	54
2	47	47
3	26	38
4	56	57
5	10	46
6	38	48
7	18	38
8	66	47
9	60	58
10	36	38
11	43	43
12	40	44
13	51	61
14	66	47
15	18	24
16	45	51
17	38	48
18	37	47
19	44	42
20	50	51
21	47	41
22	41	43
23	37	47
24	56	58
25	61	66

Try to establish a relationship between ozone concentration and AQI index by fitting a model.

Worksheet for Exercise 12

$\sum X =$, $\sum Y =$, $\sum X^2 =$, $\sum Y^2 =$

$\sum XY =$, $n =$

Model : $Y = A + BX$

Normal equations are :

$\sum Y = nA + B \sum X =$

$\sum XY = A\sum X + B\sum X^2 =$

Solve the above two equations to get values of A and B.

ANOVA

Source	Degrees of freedom	Sum of Squares	Mean Square	F ratio
Due to regression	1			
Error	23			
Total	24			

EXERCISE 13

In a particular town two samples of ambient air are tested every hour for 20 consecutive hours AQI calculated for the samples are given below s :

Area Code Number	Test Values			
	1	**2**	$\overline{X}$	**R**
1	21	20		
2	24	23		
3	20	21		
4	19	27		
5	23	18		
6	22	21		
7	19	21		
8	24	17		
9	25	23		
10	21	23		
11	18	20		
12	23	19		
13	24	25		
14	29	24		
15	26	30		
16	20	26		
17	19	20		
18	25	21		
19	19	26		
20	27	19		

1. Calculate the control limits for $\overline{X}$ - R charts
2. Draw the control charts
3. Interpret the control charts and identify the outliers.

Worksheet for Exercise 13

Overall average $\overline{\overline{X}} = \frac{}{20} =$ and $\overline{R} = \frac{}{20} =$

<u>Control Limits R – Chart</u>

Upper Control Limit (UCL) = $D_4 \; \overline{R} =$

Center Line = $\overline{R} =$

Lower Control Limit (LCL) = $D_3 \; \overline{R} =$

<u>Control Limits for $\overline{X}$ chart</u>

Upper Control Limit (UCL) = $\overline{\overline{X}} + A_2 \; \overline{R} =$

Center Line = $\overline{\overline{X}} =$

Lower Control Limit = $\overline{\overline{X}} - A_2 \; \overline{R} =$

EXERCISE 14

Samples of water were collected from different parts of Delhi. They were tested for presence of pathogens by taking a drop of water on a slide and the no. of pathogen present in the drop was counted. Check whether the counts of pathogen presence was influenced by some external reasons in any area or not. Data is as follows :

Delhi Area code number	Count of pathogen in a drop (c)
1	5
2	7
3	8
4	4
5	0
6	2
7	4
8	0
9	6
10	9
11	1
12	2
13	4
14	5
15	9

Worksheet for Exercise 14

$$\overline{C} = \frac{\sum C}{15} =$$

Center line = $\overline{C}$ =

Upper Control Limits (UCL) $= \overline{C} + 3\sqrt{\overline{C}}$

Lower Control Limits (LCL) = $= \overline{C} - 3\sqrt{\overline{C}}$

EXERCISE 15

Water samples from different parts of the country was collected. 10 samples were collected from each of 20 states. In these ten samples some were found unfit for drinking. Identify the states which reported abnormally high unfit drinking water quality.

State code	Sample size	No. of unfit samples
1	10	3
2	10	4
3	10	2
4	10	1
5	10	5
6	10	7
7	10	9
8	10	4
9	10	1
10	10	2
11	10	6
12	10	7
13	10	0
14	10	4
15	10	7
16	10	5
17	10	1
18	10	2
19	10	4
20	10	5
Total		

Worksheet for Exercise 15

Proportion of unfit drinking samples

$$= \frac{\text{Total no. of samples of unfit drinking water}}{10 \times 20} = \bar{p} =$$

Center line = n $\bar{p}$ = 10 $\bar{p}$ =

Upper Control Limit (UCL) = $n\bar{p} + 3\sqrt{n\ \bar{p}\,(1-\bar{p})}$

Lower Control Limit (LCL) = $n\bar{p} - 3\sqrt{n\ \bar{p}\,(1-\bar{p})}$

Your Comments:

EXERCISE 16

With the same data as given above check whether the proportion of unfit water is alarmingly high in any state or not.

State code	Sample size (n)	No. of unfit samples (D)	Proportion of unfit sample (D/n) = p
1	10	3	
2	10	4	
3	10	2	
4	10	1	
5	10	5	
6	10	7	
7	10	9	
8	10	4	
9	10	1	
10	10	2	
11	10	6	
12	10	7	
13	10	0	
14	10	4	
15	10	7	
16	10	5	
17	10	1	
18	10	2	
19	10	4	
20	10	5	
Total			

Worksheet for Exercise 16

Proportion of unfit drinking sample

$= \frac{\text{Total no. of proportion of unfit drinking water}}{20} = \overline{p}$

$$= \frac{\sum p}{20} =$$

Center line = $\overline{p}$ =

Upper Control Limit (UCL) = $\overline{p} + 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}}$

Lower Control Limit (LCL) = $\overline{p} - 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{n}}$

Your Comments:

STATISTICAL TABLES

TABLE A : AREAS UNDER THE STANDARD NORMAL CURVE

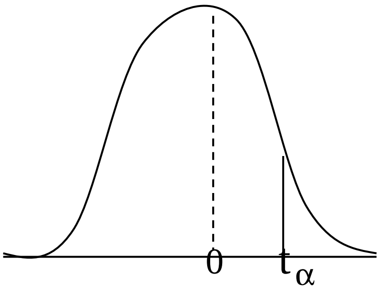
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4217
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A: continued

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5010	0.5080	0.5120	0.5160	0.5199	0.5219	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6180	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9839	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9981	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE B t – **DISTRIBUTION**

Critical Values of the *t* Distribution α



v	α 0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

Table C : Percentage Points of the F distribution ($F_{.05, v_1, v_2}$)

V_1\v $_2$	Degrees of freedom for the numerator (v_1)																	
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.0	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9 0	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.33	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1 79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1 93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2 .24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1 58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Table D : Percentage of Points of the F Distribution ($F_{.10,v_1,v_2}$)

$V_1 \backslash V_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2 .16
10	3.29	2.92	2.73	2.61	2.52	2.46	2. 41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2 .61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	1.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.67	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1 .96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	199	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1. 67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1 87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

Table E : Statistical Constants for $\overline{X}$ and R Control Charts

n	A_2	D_3	D_4	d_2
2	1.880	0	3.268	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078